UNIT-V

# WORK ENERGY PRINCIPLE.

**Work :-** Work is the product of the force applied to an Object and the displacement of the Object.

$$\boxed{W = Fd}$$

Units :- Joules (or) N-m.

**Energy :-** Capacity to do work

Units :- Joules.

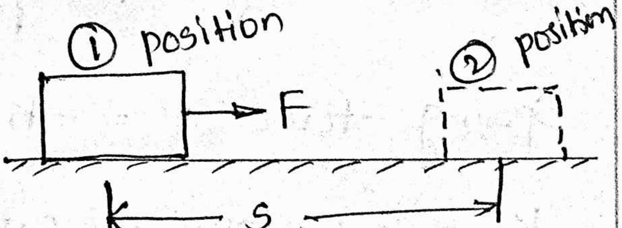
**Power :-** Rate of doing work is called power

Units :- N-m/sec (or) J/sec.

power = work/Time.

**Work done by a force :-**

If a particle is subjected to a force 'F'. and particle is displaced by 's' from position ① to position ② then

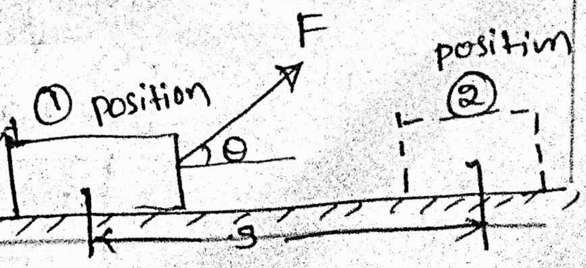



Workdone = force × Displacement.

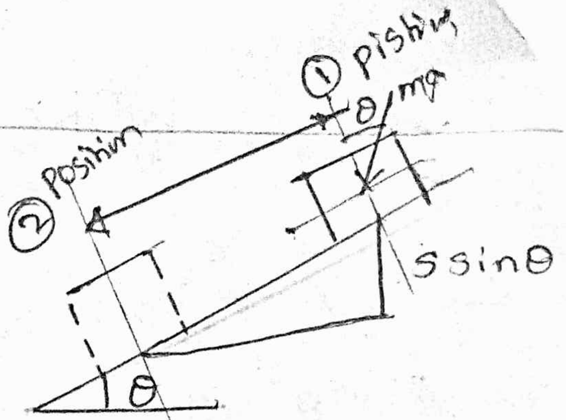
$U = F \times S.$

W.D = Component of force × Displacement

$U = F \cos\theta \times S.$



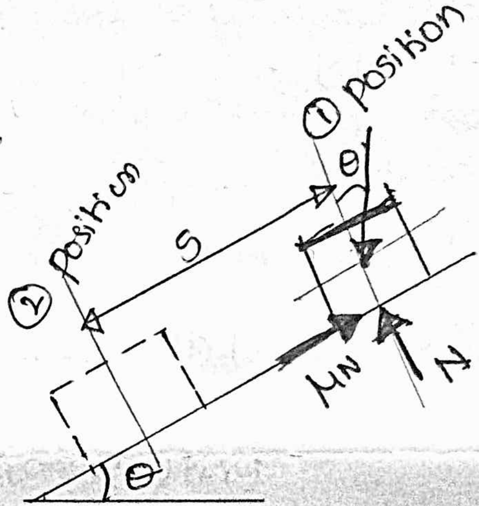
Work done by weight force.




Workdone = component of weight in the direction of displacement × Displacement.

$$U = mg \sin\theta \times s.$$

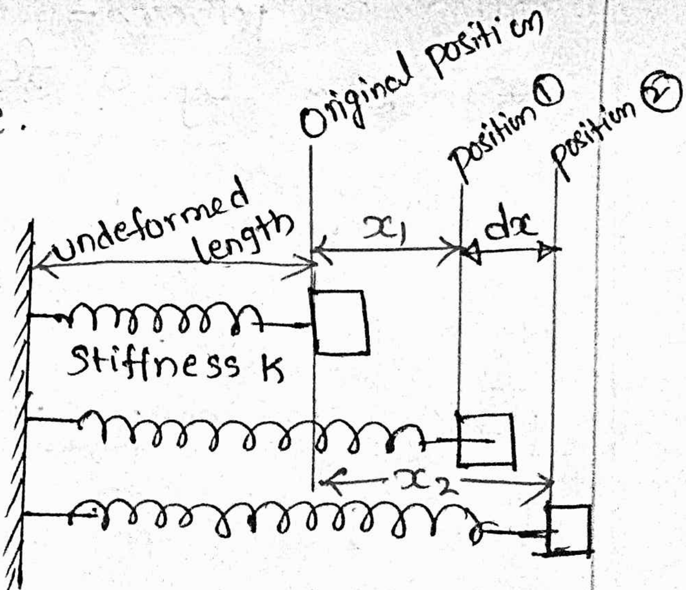
Workdone by friction force.




Workdone = − frictional force × Displacement

$$U = -\mu N \times s.$$

Workdone by spring force.




Spring force $F = -K \times x$

$K \rightarrow$ Spring stiffness (N/m)

$x \rightarrow$ deformation of spring (m)

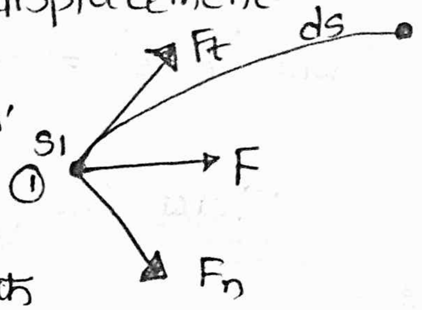
Workdone = Spring force × Deformation.

$$U = \int_{x_1}^{x_2} -Kx\,dx \quad \therefore \quad U = -\tfrac{1}{2}K(x_2^2 - x_1^2)$$

$$\boxed{U = \frac{1}{2}K(x_1^2 - x_2^2)}$$

# Work Energy principle.

**Work Energy principle** :- Workdone by the forces acting on a particle during some displacement is equal to the change in kinetic energy during that displacement.




Consider the particle having mass 'm' is acted upon by a force 'f', and ~~acting~~ moving along a path which can be rectilinear (or) curvilinear.

Let $v_1$ & $v_2$ be the velocities of the particle at position ① & position ②. and the corresponding displacement $s_1$ & $s_2$ respectively.

$$\Sigma F_t = ma_t$$

$$F\cos\theta = ma_t = m\frac{dv}{dt}.$$

$$F\cos\theta = m\cdot\frac{dv}{ds} \times ds/dt.$$

$$F\cos\theta = mv \times dv/ds.$$

$$F\cos\theta\, ds = mv\,dv$$

Integrating both sides.

$$\int_{s_1}^{s_2} F\cdot\cos\theta\, ds = \int_{v_1}^{v_2} m\cdot v\cdot dv$$

$$\therefore U_{1-2} = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2.$$

workdone = change in kinetic energy.

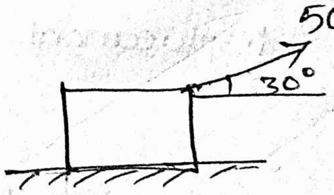
## Principle of conservation of energy

When a particle is moving from position 1 to position 2 under the action of only conservative forces then by energy conservation principle total energy remains constant.

Total Energy = Kinetic energy + potential Energy + Spring energy

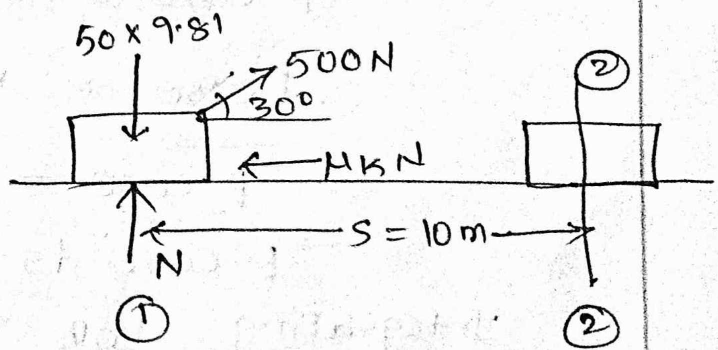
$$\text{Total Energy} = \frac{1}{2}mv^2 \pm mgh + \frac{1}{2}kx^2$$

A force of 500 N is acting on a block of mass 50kg resting on a horizontal surface. Determine the velocity of the block, has travelled a distance of 10 m. Take $\mu = 0.5$.




$$\Sigma F_{ay} = ma_y$$

$$(\because a_y = 0)$$




$$N_0 - 50 \times 9.81 + 500 \sin 30^\circ = 0$$

$$N = 240.5\,N$$

By work Energy principle.

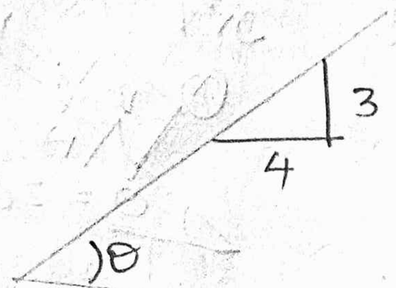
Total work done = change in Kinetic Energy

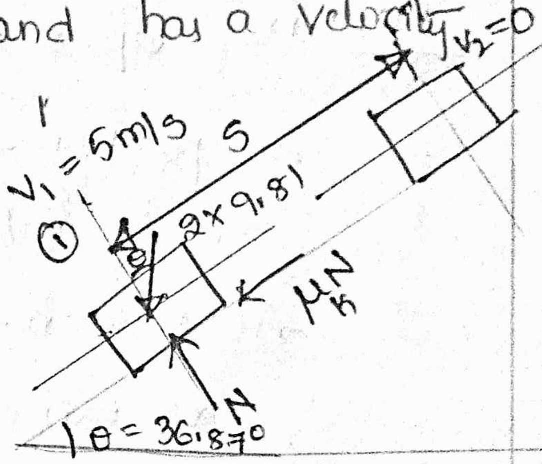
$$500 \cos 30^\circ \times 10 - \mu N \times 10 = \frac{1}{2} \times 50 \times V_2^2 - 0$$

$$500 \cos 30' \times 10 - 0.5 \times 240.5 \times 10 = 25\,V_2^2$$

$$\boxed{V_2 = 11.185\ m/s}$$

Block A has a mass of 2kg and has a velocity of 5 m/s up the plane. use




By principle of work energy.

workdone = change in K.E.

$$-2 \times 9.81 \sin\theta \times S - \mu N \times S = 0 - \frac{1}{2} \times 2 \times 5^2$$

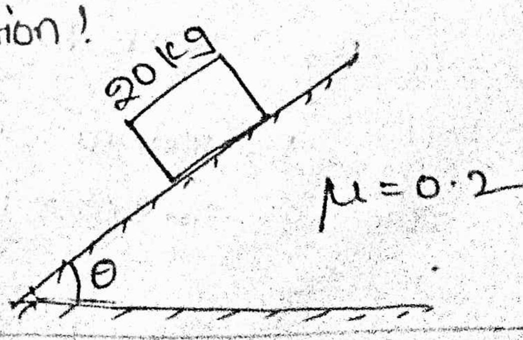
$$-2 \times 9.81 \sin 36.87^\circ \times S - 0.2 \times 2 \times 9.81 \cos 36.87^\circ \times S$$
$$= -\frac{1}{2} \times 2 \times 5^2$$

$$S = 1.68 \text{ m.}$$

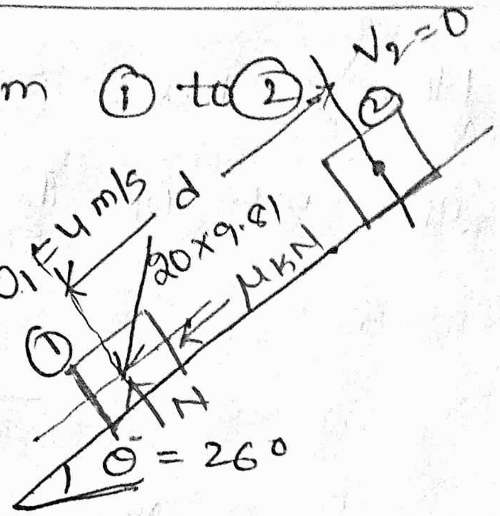
A mass of 20 kg is projected up an inclined of 26° with velocity of 4 m/s. If $\mu = 0.2$.

i) find the maximum distance that the package will move along the plane

ii) What will be the velocity of the package when it comes back to Initial position!



Case i, Upward motion from (1) to (2)




At position – (1)

$v_1 = 4$ m/s.

At position – (2)

$v_2 = 0$

By work Energy principle

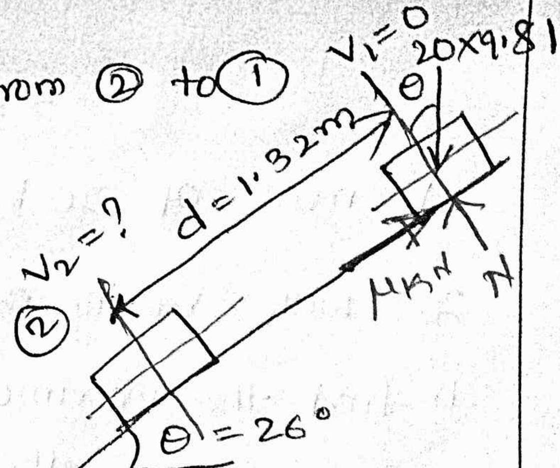
Total Work done = change in K.E.

$$-20\times 9.81 \sin 26° \times d - 0.2\times 20\times 9.81 \cos 26° \times d$$
$$= 0 - \frac{1}{2}\times 20\times 4^2$$

$$d(86 + 35.37) = 160$$

$$\boxed{d = 1.32\text{ m}}$$

Case ii, Downward motion from (2) to (1)




At position – (1)

$v_1 = 0.$

At position – (2)

$v_2 = ?$

Displacement $d = 1.32$ m.

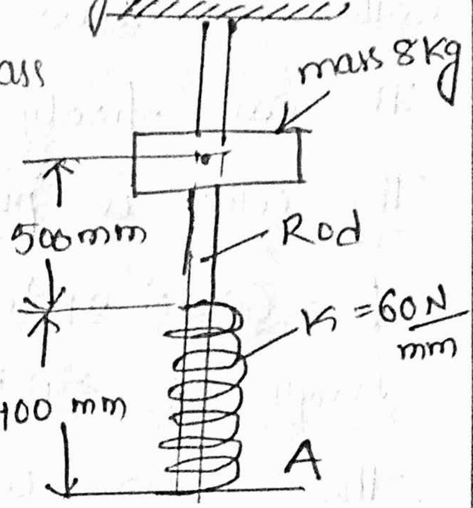
By work Energy principle.

workdone = change in K.E

$$20\times 9.81 \sin 26° \times 1.32 - 0.2\times 20\times 9.81 \cos 26° \times 1.32$$
$$= \frac{1}{2}\, 20 v_2^2 - 0$$

$$\boxed{v_2 = 2.59\text{ m/s}}$$

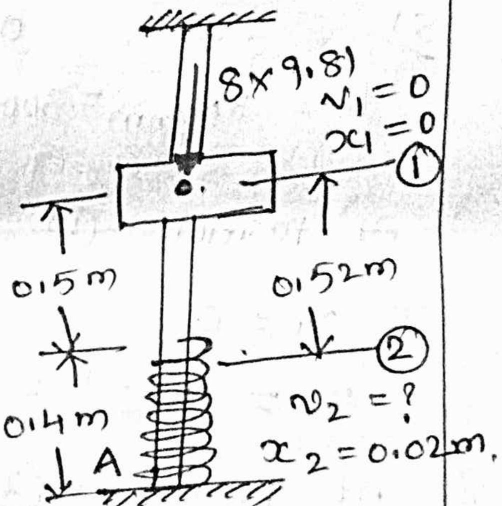
A block of mass 8 kg slides freely on a smooth vertical rod. The mass is released from rest at a distance of 500 mm from the top of the spring. The spring constant is 60 N/mm. Determine the velocity of block when spring compressed through 20 mm. The free length of the spring is 400 mm.




$K = 60\ N/mm = 60000\ N/m$

At position — 1

$v_1 = 0,\ x_1 = 0$




At position — 2

$v_2 = ?,\quad x_2 = 0.02\ m$

Total displacement $= 500 + 20$

$= 520\ mm$

$S = 0.52\ m.$

By work Energy principle.

Total W.D = change in K.E.

$$8 \times 9.81 \times 0.52 + \frac{1}{2} \times 60000\,(0 - 0.02^2) = \frac{1}{2} \times 8 \times v_2^2 - 0.$$

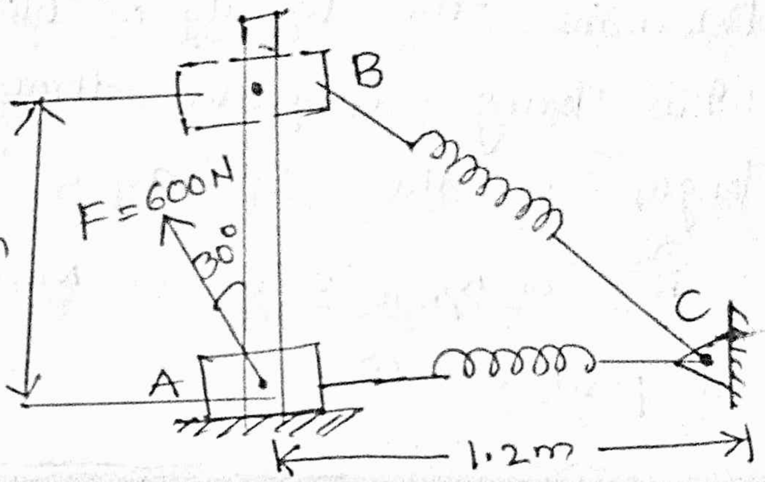
$$28.8096 = 4 v_2^2$$

$$\boxed{v_2 = 2.684\ m/s}$$

Collar of mass 15 kg is at rest at 'A' It can freely slide on vertical smooth rod AB. The collar is pulled up by with a constant force F = 600N applied as shown in fig. Unstretched length of spring is 1m. Calluclate velocity of the collar when It reaches position B.

Spring constant.

K = 3 N/mm




Sol

$K = 3 N/mm = 3000 \frac{N}{m}$

At position - ①

$v_1 = 0$

$x_1 = 1.2 - 1 = 0.2 m$

At position - ②

$v_2 = 0$

$x_2 = (BC) - 1 = 1.5 - 1 = 0.5 m$

By work energy principle

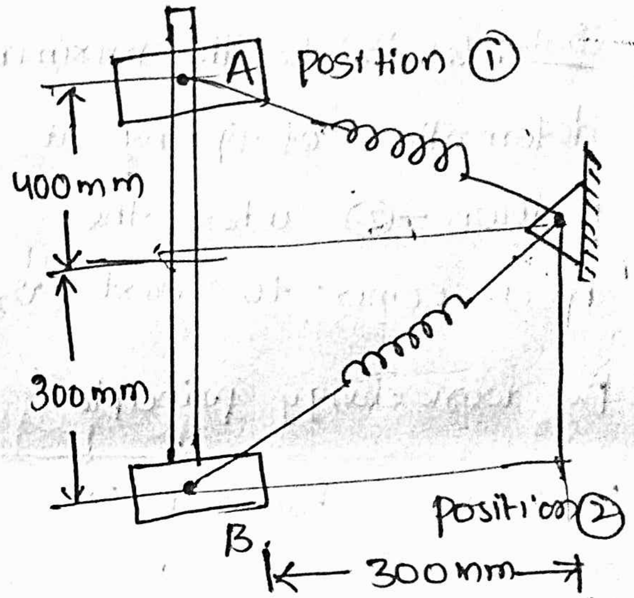
workdone = change in K.E

$600 \times \cos 30° \times 0.9 - 1.5 \times 9.81 \times 0.9 + \frac{1}{2} 3000 (0.2^2 - 0.5^2)$

$= \frac{1}{2} \times 15 \times V_2^2 - 0$

$\boxed{V_2 = 1.64 m/s}$

A collar A of mass 10 kg moves in vertical guide as shown in fig. Neglecting the friction between the guide and the collar, find. It's velocity when it passes through position ② after starting from rest in position ①. The spring constant is 20 N/m and the free length of spring is 200 mm.




<u>Sol</u> $k = 200\,N/m$

Free length of spring = 200 mm

= 0.2 m

At position – ①

$x_1 = 500 - 200 = 300$ mm

$x_1 = 0.3$ m

$v_1 = 0$

At position ②

$x_2 = 424.26 - 200 = 224.26$ mm

$x_2 = 0.224$ m

$V_2 = ?$

By work Energy principle

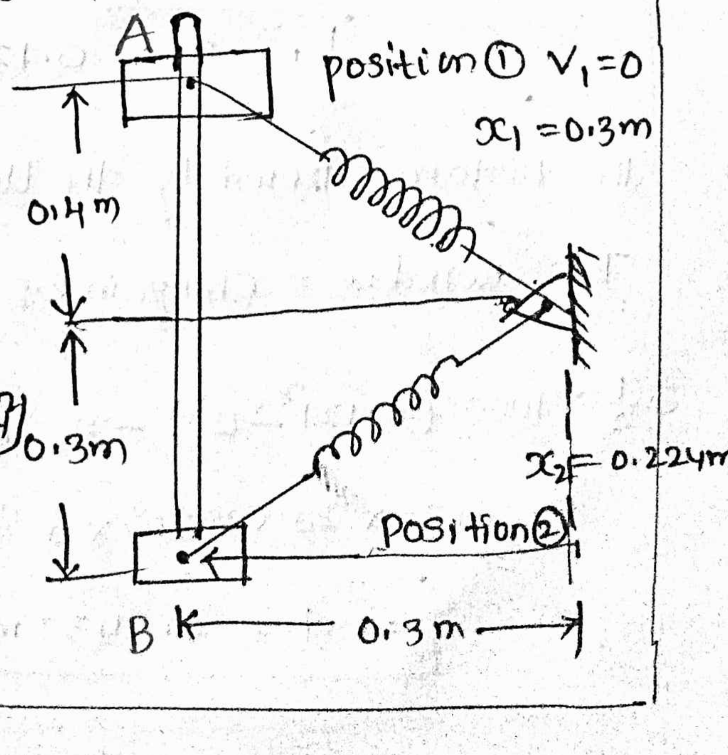
Total work done = Change in K.E

$10 \times 9.81 \times 0.7 + \frac{1}{2} \times 200\,(0.3^2 - 0.224^2)$

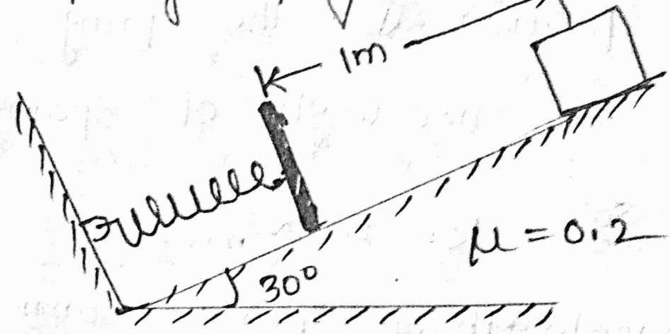
$= \frac{1}{2} \times 10 \times V_2^2 - 0$

$72.65 = 5 V_2^2$

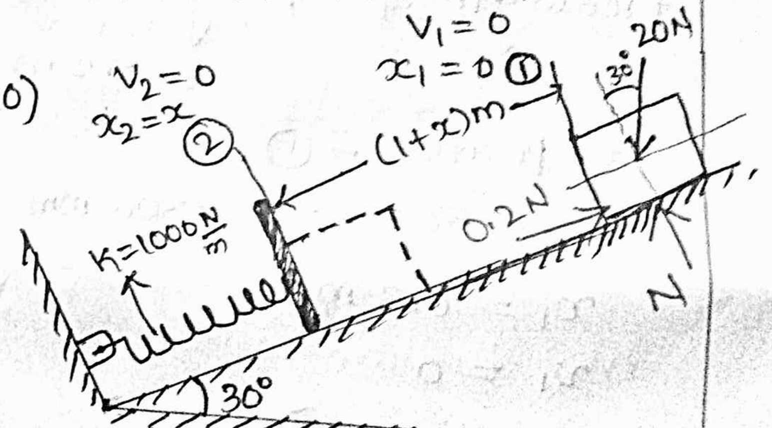
$\boxed{V_2 = 3.81\ m/s}$



A 20 N block is released from rest. It slides down the inclined having $\mu = 0.2$ as shown in fig. Determine maximum Compression in the Spring and the distance moved by the block when the energy released from Compressed Spring. Spring Constant is $K = 1000$ N/m.




Sol: let 'x' be the maximum deformation of spring at position – ② where the block comes to rest ($v_2 = 0$)




By work energy principle

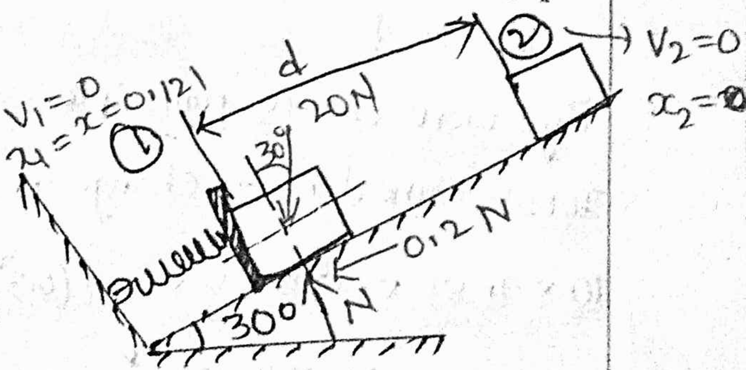
workdone = Change in K.E

$$\frac{1}{2} \times 1000 (0^2 - x^2) + 20 \sin 30^\circ (1+x) - 0.2 \times 20 \cos 30^\circ (1+x) = 0 - 0$$

∵ final & intial velocities are zero.

$$\boxed{\therefore x = 0.121 \text{ m}}$$

ii) Distance moved by the block




Total workdone = Change in K.E

$$\frac{1}{2} \times 1000 (0.121^2 - 0^2) - 20 \sin 30^\circ \times d - 0.2 \times 20 \cos 30^\circ \times d = 0 - 0$$

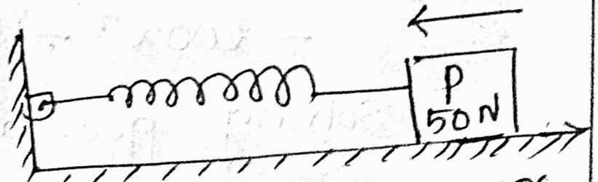
$$\boxed{\therefore d = 0.5437 \text{ m}}$$

A block 'p' of weight 50 N is pulled so that the extension in the spring is 10 cm. The stiffness of the spring is 4 N/cm & the co-efficient of friction b/w the block & the plane o-x is $\mu = 0.3$. find

i) Velocity of the block as the spring returns to its undeformed state

ii) The maximum compression in the spring

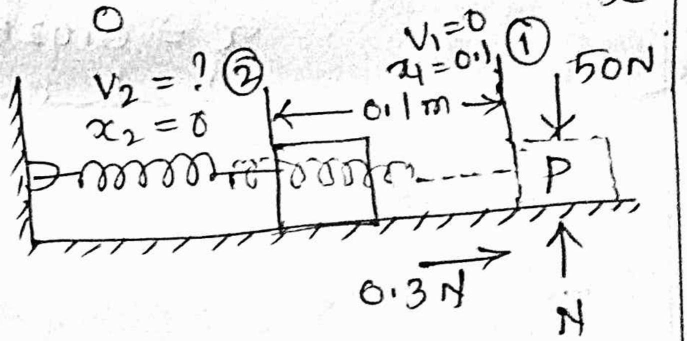
$K = 4 \text{ N/cm} = 400 \text{ N/m}$




i) At position – 1

$V_1 = 0$

$x_1 = 10 \text{ cm} = 0.1 \text{ m}$




ii) At position – 2

$V_2 = ?$

$x_2 = 0.$

from work energy principle.

Displacement = 10 cm = 0.1 m.

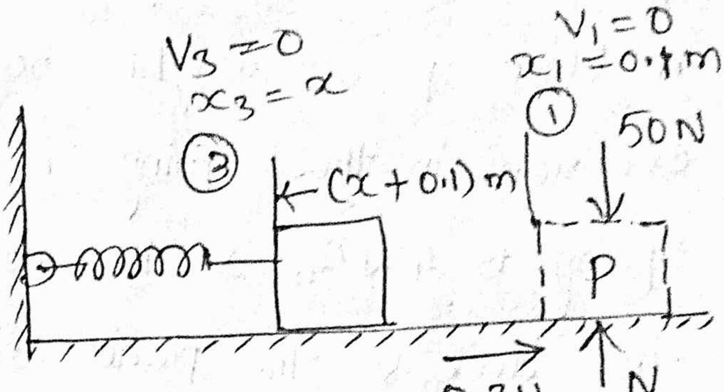
workdone = change in K.E

$-0.3 \times 50 \times 0.1 + \frac{1}{2} \times 400 (0.1^2 - 0^2) = \frac{1}{2} \times \frac{50}{9.81} V_2^2 - 0$

$0.5 = 2.548 V_2^2$

$V_2 = 0.44 \text{ m/s} (\leftarrow)$

ii, At position — 2




$V_3 = 0$

$x_3 = x$.

Appling work energy principle from position 1 to position 2

$$-0.3 \times 50(0.1 + x) + \tfrac{1}{2} \times 400 \left(0.1^2 - x^2\right) = 0$$

$$-1.5 - 1.5x + 2 - 200x^2 = 0$$

$$-200x^2 - 15x + 0.5 = 0$$

Solving quadratic equation, we get

$$x = 0.025\,m,$$